On p. 147, in speaking of an observation on after-images, it is said that "they disappeared gradually, but in the spot where they had vanished from the *orbit* of the closed eye," &c. Here the *field* of vision seems to be intended. When we read (p. 107) of "the *marginal* elements to which we are led by analysis of compound states of consciousness," the *ultimate* elements are presumably intended. Is it in accordance with Prof. Höffding's original Danish that the corpora striata and optic thalami are placed in the mesencephalon? These are but slight and scattered blemishes in an excellent piece of work.

C. Ll. M.

THE FOUNDATIONS OF GEOMETRY.

The Foundations of Geometry. By E. T. Dixon. (Cambridge: Deighton, Bell, and Co., 1891.)

"OUGHT there not to be a perfect subjective geometry, as well as an applied objective one, the applicability of the former to the latter being a matter to be determined by induction from experiment? It is the object of this book to show that such is the case, to establish the perfect geometry, and to examine the grounds on which we may believe that it applies to the objective space in which we live." Such, stated in the author's own words, is the object he has proposed to himself, and he appeals with confidence to geometricians to criticize his system. The work is divided into three parts, devoted respectively to (1) a discussion on the logical status of geometry; (2) the development of the author's subjective theory; and (3) an investigation into its application to material space. It is to part (2) that we shall chiefly direct our remarks; for though the author's views, as laid down in part (1) (see pp. 20, 21), on definition as the basis of a deductive system, have an important bearing on his geometrical theory, and seem in opposition to those of at least one school of logicians, they will be best tested by an examination of the definitions of the two concepts "direction" and "sameness of direction" laid down at the beginning of part (2). With much of the author's criticism of Euclid's treatment of the plane we are in agreement, but he seems to lay himself open to the chief objections urged against his predecessors. With regard to part (2), *granting the assumptions tacitly made at the outset*, we are willing to admit the formal accuracy of most of the proofs of the propositions in the "subjective theory," and the possibility of supplying it, without any serious derangement of the system, to those which seem to want it. But we do take serious exception to the way in which the foundations of the new edifice are laid. The system is based partly on three axioms, explicitly stated, as to (1) the possible transference of geometrical figures, (2) the possible extension of a straight line, (3) the three-way extension of space; and partly on what are styled, not inaptly, the "implicit" definitions of *position* and *direction*. With that of "position" (used in the sense of "fixed point") we are not much concerned, merely remarking that in it the word *position* is used to explain *position*. This defect, under which Bardolph's more famous definition of "accommodated" also labours, could probably be easily rectified. But so much depends on that of "direction" and "sameness of direction," that we give it in full:—

"Implicit definition of direction.

"(*a*) A direction may be conceived to be indicated by naming two points, as the direction from one to the other.

"(*b*) If a point move from a given position constantly in a given direction, there is only one path, or series of positions, along which it can pass. (Such a path may be called a 'direct path,' and a continuous series of points occupying such positions, a straight line.)

"(*c*) If the direction from A to B is the same as that from B to C, that from A to C is also that same direction.

"(*d*) If two unterminated straight lines, which intersect, are each intersected by a third straight line in two separate points, any unterminated straight line extending in the same direction as this last one, which intersects one of the two former, shall also intersect the other."

Now, we mistrust an "implicit" definition. It seems to be a compound of "axiom and definition" or "postulate and definition," which ought in each case to be carefully analyzed into its elements. As to (*a*) after repeated mental effort, we are still unable to realize the meaning of the "direction from A to B" without using the concept of the "straight line AB." Hence (*b*) seems merely to tell us that moving along the straight line from A to B may be called "moving constantly in the same direction from A to B," and to lay down the *axiom* or *postulate* that "only one straight line can be drawn from A to B," while the only additional explanation given by (*c*) is that if two different straight lines meet their directions are said to be different. As yet we have not received any information as to what is meant when two non-intersecting straight lines are said to have the "same direction" or to have "different directions." We proceed to (*d*) to find what direction from C is that which may be said to be the same as that from A to B. Following the instructions, we must imagine two straight lines OP, OQ, both of which intersect AB, and one of which passes through C; then the straight line which passes through C, and intersects OQ in some point D, such that CD does not intersect AB, however far they are both produced, is said to have the "same direction" as AB.

Surely there is a tacit assumption here *that there is one and only one straight line through C which intersects OQ, and being produced does not intersect AB;* without it the definition seems unmeaning. Yet this afterwards appears as Proposition 2. There is certainly a further one which may be enunciated thus: *If there are two non-intersecting straight lines AB, CD, such that CD meets each of two given intersecting straight lines which each meet AB, then if CD meets either of any other such pair of intersecting straight lines it meets each of them.* If this axiom be not admitted the supposed demonstration of Proposition 2 fails.

It must be clearly understood that we are not objecting to the axioms themselves as fit foundations for a logical system; they seem to be tantamount to those which Euclid either openly or tacitly assumes, or, as it would be better to say, *postulates*. We do not think, indeed, that the author has done his own method justice, for it might be simplified by laying down its αἰτήματα explicitly; and a comparison of them with those tacitly or openly

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made by Euclid in his definition of a plane, in "Axioms 10 and 12," and in XI. 1–3, would serve to clear up much of the obscurity which it must be confessed has been generally allowed in text-books to hang over this part of the "Elements." It may be incidentally remarked that in Henrici's "Congruent Figures" and Hayward's "Elements of Solid Geometry" these matters receive due discussion. It will serve as a concise summary of the points of contrast between Mr. Dixon's system and that of Euclid as elaborated by commentators to say that, whereas Euclid, starting from the concepts *straightness* and *flatness*, deduces from certain αἰτήματα as to these concepts the concept *parallelism*, Mr. Dixon, starting from the concepts *straightness* and *sameness of direction*, deduces from certain αἰτήματα as to these concepts the concept *flatness*.

While open to the objections we have urged, and to some others on the treatment of the plane angle (we do not, for instance, see any logical cogency in the demonstration of Proposition 7), this part of the work seems to have been elaborated with skill and care.

Part (3) contains some acute remarks on the possibility of a "fourth dimension," and on a method of forming diagrams in material space which shall be "orthogonal projections of the true figures."

We heartily recommend the book to the attention of all those interested in the presentation of the "Elements" to beginners, whether they are authors, teachers, or examiners. Authors may find suggestions for many a note on fundamental difficulties; teachers may pick up a frequent hint for class-work with their more intelligent pupils; and examiners might do worse than set an occasional critical question on some of the assumptions which the author reminds us it is usual to make tacitly. We notice with interest the appearance of a proof of "Axiom 12," which the author, after Prof. Newman, ascribes to M. Vincent, of Paris. A similar one occurs in Dr. Thomson's edition of the "Elements," and is said to be due to M. Bertrand, of Geneva, and it is also to be found in chapter xv. of De Morgan's "Study and Difficulties of Mathematics," with the remark that it "is more than probable that had the same come down to us sanctioned by the name of Euclid it would have been received without difficulty."

E. M. L.

OPTICAL PROJECTION.

Optical Projection. By Lewis Wright. (London: Longmans, Green, and Co., 1891.)

MR. WRIGHT has earned the warm thanks of all teachers and students who use the lantern for lecture or demonstration purposes by this excellent book. It contains about 400 pages. Of these, the first half is devoted to descriptions of the various parts of a lantern, and of apparatus accessory to its use. The principles of projection are clearly explained; then follows an account of the different forms of condensers and their relative advantages. Mr. Wright's criticisms of the various forms are clear and to the point. He recommends, as the one which is generally most useful, two plano-convex lenses with their plane surfaces turned outwards, the lens nearer the radiant being rather the smaller. A discussion of the various forms of objectives comes next, with practical hints for testing both them and the condensers. After this we have several chapters devoted to possible sources of light.

The form of jet for the lime-light which the author recommends is a mixed jet devised by himself, and described on p. 63; this, he says, can sometimes be pushed as bright as 1000 candles; while for the arc light a modification of the Brockie Pell lamp, described on p. 163, seems to have given the most satisfactory results. Full details for using these and other forms of light are given, and the possible sources of danger to which each may give rise are carefully discussed.

The book is throughout thoroughly practical. The best method of preparing the gas, the best form of gas bag, and of pressure boards, the various adjustments required in setting up the burner so as to give the best result, the details of focussing, and many other points, are considered in turn. The best screen appears to be a smooth surface of plaster of Paris, after that a whitewashed wall finished with whitening.

In chapter xii. we have an account of lanterns for special purposes, scientific demonstrations, and the like; among others the elaborate lantern used at the Royal Institution is described.

Considerable space is devoted in chapter xiii. to the projection-microscope, of which the important development in the past ten years is largely due to Mr. Wright.

This concludes the first half of the book. The second part deals with demonstrations in the various branches of natural science. Experiments are described in physics—including heat, light, sound, electricity and magnetism—chemistry, physiology, &c.

With regard to many of these a general criticism may be made. Mr. Wright, in fact, almost suggests it himself on p. 215. There may be occasions on which apparatus and experiments can be shown only by means of projection. The room may be too large and the audience too numerous to permit of the lecture-table being well seen, and the details of an experiment as it is actually performed being followed by all; or, again, a lecturer may have to travel about, as in the case of University Extension lecturers, to various centres; he can fairly easily carry with him slides and small apparatus, whereas it would be impossible to transport all that is required for the performance of the same experiments on a large scale. In such cases as these Mr. Wright's practical details will prove invaluable, but for smaller audiences, or in lecture-rooms attached to permanent laboratories, more will, we think, be learnt from seeing the same experiments performed on a larger scale, rather than with apparatus specially devised to render projection possible.

Another difficulty of a practical kind in the use of the lantern, is that the room must be darkened. Students cannot easily see to make diagrams or take notes; and the lecturer cannot see them, and discover from their faces, as he proceeds, how far he is making himself understood and is interesting the class. However, as was pointed out at the last meeting of the British Association, in an interesting paper by Profs. Barr and Stroud, on the use of the lantern, the room may for many experiments be comparatively bright, provided that the screen is well shaded from the light.

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